PERFECTLY in TUNE

Towards Intonation Correctness

by SERGE CORDIER

LECTURE

by Serge CORDIER

Lecture given during the 1991
"Choral ConductingTechniques" Summer University
in Chambon-Sur-Lac (Puy de Dôme, France)

« Actes des Universités d'été 1991-92-93 »
Editions du CRDP, 15 rue d'Amboise,
Clermont-Ferrand.

Perfectly in Tune
Towards Intonation Correctness

by Serge CORDIER

IS THE PURE FIFTH EQUAL TEMPERAMENT A NEW TYPE OF PIANO TUNING, OR A NEW THEORY ON PERFECTLY IN TUNE - WHETHER FOR PIANO, VOICE OR ORCHESTRA?

The lecture I delivered on the 5th September1991 at the Chambon-sur-Lac Summer University forms part of a series of lectures to present a new form of tuning : the "Pure Fifth Equal Temperament" or PFET. Indeed, we know that the traditional piano tuning called the "tempered scale" or the "well-tempered scale" is based on the temperament of the fifth, i.e. on altering that interval, and that traditional theory postulates that it is impossible to maintain pure fifths on a keyboard instrument, that is, if one aims to access every tone whilst limiting to 12. the number of notes per octave.

Yet this is perfectly possible and, judging by the quasi-unanimously favourable reactions to this form of tuning * ***footnote 1***, it even proves to be a far better solution than that of shortening the fifths to obtain the same result. The members of the audience at the Summer University were indeed able to judge this for themselves after hearing June Pantillon play a sonata by Scarlatti on a piano I had previously tuned to PFET * ***footnote 2*** .

Now, two obvious questions immediately spring to mind : why does this tuning sound more perfect than the tuning known traditionally as "the tempered scale", and why, since it is possible to tune in this way, was it not discovered and applied earlier ?

Before answering these, and to set out the problem clearly, I wish to draw my listeners′ attention to another, no less significant, fact : for the past few decades, both acousticians and musicians have noticed that the range of frequencies of a well tuned piano revealed noticeable differences from the range of frequencies of the tempered scale - the latter, may I remind you, being the result of dividing the perfect octave into 12 equal semi-tones * ***footnote 3*** . Good tuners - according to E. Leipp in "Acoustique et Musique", Edition Masson, 1988 - do not tune in a "tempered" way, at least not in the traditional sense of the term. Conversely, he goes on to say, a piano that has been tuned strictly in accordance with the tempered scale, using electronically set frequencies, sounds "completely out of tune".

Though I myself would be inclined to suggest that E. Leipp's statements need not be taken at face value, it does remain true to say that a piano tuned the PFET way unfailingly sounds better and more musical than a piano tuned to the frequencies of the tempered scale. This then means that my concept of tuning the PFET way matches the apparently abnormal practice in use by good piano tuners far better than the traditional theory of the tempered scale does. But, once I had noted and accepted that fact, I did not want to stop at that, and I researched the reasons for the superior quality in the realm of better pitch correctness and musicality,of PFET over the tempered scale, - and this brings us back directly to the unavoidable questions raised above, i.e. : why does a piano tuned to pure fifths sound better than one tuned traditionally in tempered fifths, and why, - if this way of tuning sounds so good - , have we been waiting for Cordier to discover it ?

Before calling on essentially musicological and cultural arguments (which, contrary to expectations, do not arise from the physics of acoustics or physiology) in order to explain the success of such a way of tuning, I shall firstly begin by answering the second of these questions, namely : why have we waited for Cordier to discover it ? I shall point out first of all that, really speaking, people did not wait for me to put it into practice, or at least get close to it, as is testified by numerous remarks made by musicians or acousticians concerning the perceived gap between the theory of the tempered scale and the actual practice of good tuners. My guess is that, quite simply, as is often the case in the field of music writing (whether of harmony or counterpoint, for instance), here, practice had preceded the theory that I have formulated.

In other words, I have probably not invented PFET any more than Christopher Columbus invented America because, just like America, PFET already existed ! I simply uncovered it beneath the tuning hammer of the best tuners, - and here I must pay homage to my master Simon Debonne, the outstanding tuner of one of the great Paris piano houses - , and I went on the recognise it also beneath the bow of violinists and other stringed - instrumentalists, which is where I believe it was born ! No doubt as long as 2 centuries ago, when the equal temperament became widespread, and AS A PRACTICAL RESPONSE TO THE NEW NEED THUS GENERATED, musicians and tuners invented PFET. Like Christopher Columbus, all I had to do was to recognise it and then explore it.

Before inviting my listeners to explore this musical land which is as ancient in its practice as it is new in our awareness, we do however need to resolve one last enigma : how come, if such a temperament already existed - at least potentially - in practice, that we had to wait for Cordier to become aware of it and formulate its theory?

I asked myself this question for a long time without being able to come up with a satisfactory answer. There is a first answer to it : in order to become aware of the existence of such a temperament, one surely needed to be not only a tuner but also a musician and acoustician. However, tuners have - at best - but a paltry knowledge of acoustics ; that is why most of them are totally ignorant of the principle which explains why one should tune the fifths on a piano, and do not even know what exactly a “tempered fifth” or the “tempered scale” actually is… In this field, most musicians are no better off than they are… Such matters are examined in depth in the classes on Tuning which the Direction de la Musique asked me to give at the Montpellier Conservatoire… As for acousticians when they are musicians, that does not mean they are in any way professional tuners, nor do they possess the extraordinarily finely tuned ear of those piano technicians. Luckily they have recently acquired high-precision electronic instruments that are a tremendous aid to research ; yet these can only partially replace the ear of a musician and especially that of the piano tuner. It must be said that the specialist i.e. the musician - is somewhat loath to don the artisan's apron, which explains that I have on occasion come across a number of fools who refused to take my findings into account because I was a piano tuner ! Yet had I not been one, I would never have been able to hear the deep subtleties of the temperament and understand why for so long the true problem had gone unnoticed : the explanation lies to a large extent in the tuning techniques employed in the past but also to the erring of theoricians - i.e. acousticians - who based themselves on mathematical or philosophical “a priori” that had little to do with the art of music.

The key to this enigma was finally delivered to me by the study of the conditions under which the equal temperament progressively emerged at the beginning of the 18^{th} century.

We know that, during the Middle Ages, keyboard instruments - the organ, mainly - were tuned following a Pythagoric approach, in which each fifth was a true fifth- save one called the “wolf fifth”, being impracticable because it is short by a comma, sounds very false, and is therefore impossible to play. So why was this fifth so false and where did this troublesome comma spring from ? To understand, we need to remember that in our Western music all the notes can be *linked* in fifths or - which comes to the same thing - in fourths. Thus, as Pythagoras demonstrated 6 centuries BC, the 7 notes of the diatonic scale :

do re mi fa sol la si do

(the antique Greek scale was both modal (the *Mi* mode) and descending, but here I am deliberately simplifying these historical details to avoid unnecessary complications)

may be regarded as the result of suite of 6 successive fifths starting from an original sound which for convenience's sake we shall arbitrarily call *fa,* without attributing to this term any notion of absolute pitch - (a relatively modern notion) :

fa do sol re la mi si

If, as was the case in Antique and Mediaeval times, we maintain all of these pure fifths (or fourths), we obtain a so-called "Pythagoric" scale composed of commas ***footnote 4*** as follows :

Do re mi fa sol la si do

9c 9c 4c 9c 9c 9c 4c

We observe that this scale possesses 2 diatonic semi-tones to a value of only 4 commas, which is less than half a Pythagoric tone (9c). So, Pythagoric diatonic semi-tones are short, and this observation, together with the use of perfect fifths, lies at the origin of the debateable claim that violins and stringed-instruments in general are played according to Pythagoric scales.

At this stage of the explanation, we need to make a very important distinction - as we shall see - between the Pythagoric theory of generation of the scale by successive pure fifths, and the actual practice of Pythagoric tuning, such as was applied no doubt in Antiquity, and quite certainly in the Middle Ages.

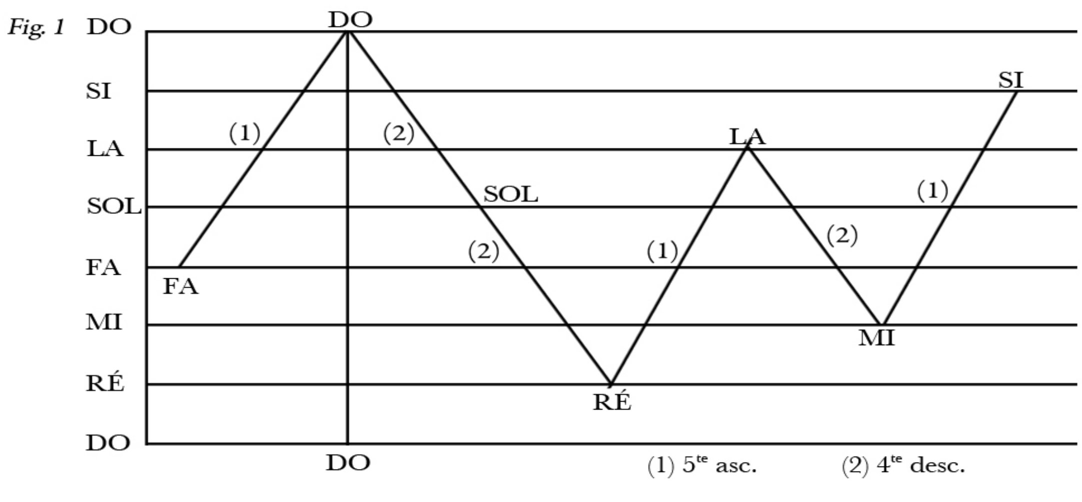
Pythagorean theory states that the notes give rise one to another, from fifth to fifth, following this pattern :

fa do sol re la mi si

But if we proceed in this way we do not obtain a scale! To do that, we then have to group together the resulting notes within a single octave (from the *do* to the *do* immediately above, for instance, if we want to make a scale in *do*) and proceed by jumping from one octave to the next. Thus **in theory** we need to perform two operations : - producing the notes from fifth to fifth, and then transferring these notes within a single octave (from one *do* to the *do* above, for example) in order obtain a linked scale such as the scale of ***do***.

In practice - and it's what tuners still do today - one manages to produce the required notes directly within the same octave by replacing an ascending fifth by a descending fourth every time one is on the point of overshooting the chosen octave.

Figure 1



The same is true of the first Western chromatic 12-tone scale found on keyboards right from the C13th, i.e.

Do do# re re# mi fa fa# sol sol# la sib si do

Which may be regarded as the result of a succession of 12 fifths stretching from bass to treble over about 7 octaves

Sib fa do sol re la mi si fa# do#sol# re# sib

But if, in tuning in this way we maintain the 11 first pure fifths as they did in the Middle Ages, then the last fifth in the cycle, i.e. *re#/sib*, will then be a fifth missing a comma : the medieval "wolf fifth". Why is it missing a comma ?

Let us suppose that we indeed wish to prolong the cycle beyond that *re#,* maintain a pure fifth *re#/ la#,* we would have :

Sib fa do sol re la mi si fa# do# sol# re# la#

However, this final *la#* would not at all merge with the *sib* which lies 7 natural octaves above the starting *sib,* but would in fact be one comma above it. This is explained by simple arithmetic : when one starts from the first note of the cycle, *sib*, and goes right up to the last *la#* one has indeed spanned 12 pure fifths ; that is to say, since a pure fifth is worth 31 commas :

31 x 12 = 372 commas.

Now, a pure octave is worth 53 commas, and this means that the *sib* which lies 7 octaves above the initial *sib* is only

53 x 7 = 371 commas away (see fig 3)

As a result, if one wants to limit the number of notes per octave to 12 and avoid creating a 13th note - *la# -,* a pure fifth above the 12th note *re#,* then the ultimate fifth *re#/sib* will well and truly be missing a comma since *sib* is a comma lower than the *la#* (that we avoid using so as not to have to add a 13th key onto the keyboard). Indeed, should we give in to that temptation, we should also have to add a 14th note - *mi#* - as distinct from *fa,* to avoid giving rise to another a new wolf-fifth, this time placed between *la#* and *fa*, and so forth, merely pushing the wolf-fifth up from one fifth to the next.

All these difficulties arise from the fact that the sum of the commas in 12 pure fifths exceeds the sum of commas in 7 octaves by the Pythagorean comma, as we have seen !

In other words, it is impossible to play all the tones with only 12 sounds per octave if one wishes to preserve both all the natural (ie acoustically perfect) fifths and the natural octaves.

If one did keep them, not only would the enharmonic notes such as *la#* and *sib* for instance not be one and the same but separated by one comma, but also one of the fifths , the 12th in the cycle, would then found to be missing one comma and therefore impossible to play.. (i.e the one which lies between the 12th and last note of the cycle - *re#* in the example above - and the 1st note, here, *sib*, taken as the 13th note one octave above the 1st).

In fact it is highly likely that people became aware of this fact without making precise calculations, but - and this is the main point - by regarding it as the natural consequence of a particular type of tuning technique which, as we have seen for the diatonic scale, does not consist in beginning piling up the fifths over an ambit of 12 fifths, and then assembling the notes by related degrees within a single octave (no instrument in mediaeval times would have allowed this) but in obtaining the 12 notes within one octave from the outset**. This was done by replacing one ascending fifth by a descending fourth** every time one was on the point of overshooting an octave.

This is what piano tuners call "setting the bearings" or making the "temperament octave", that is to say, dividing up a selected octave.

Thus, in order to directly obtain the following Pythagoric cycle

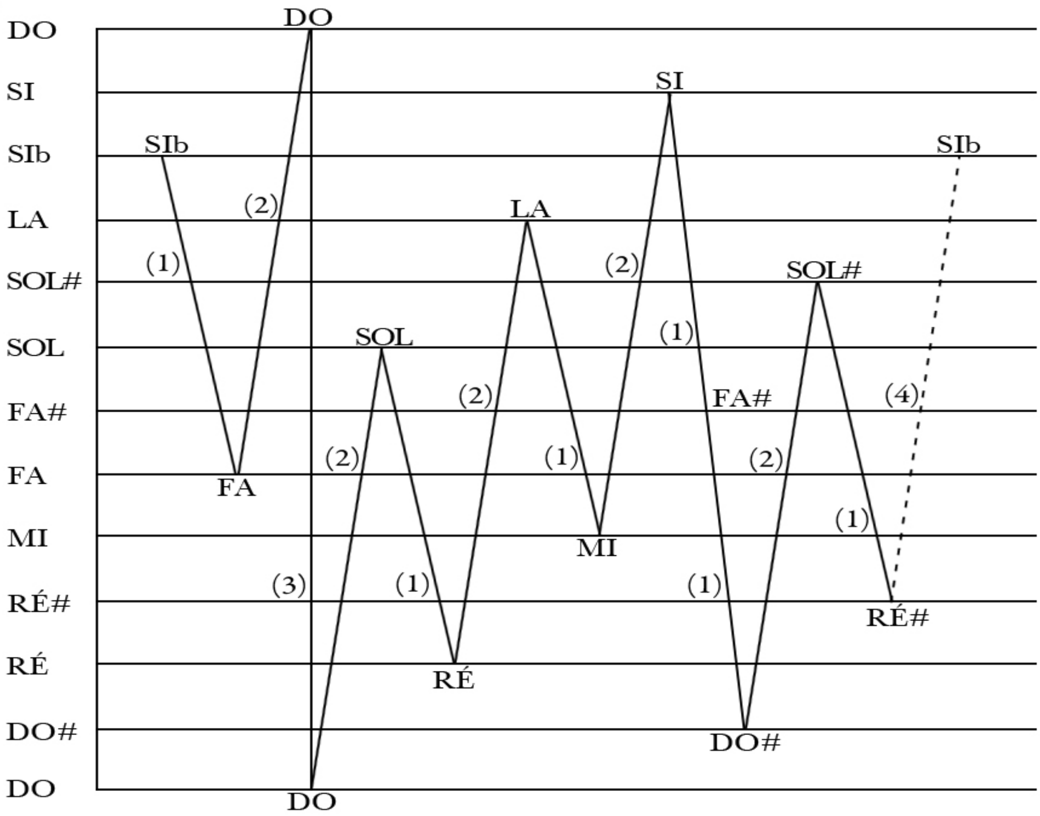
Sib fa do sol ré la mi si fa# do# sol# re# sib

in the form of a chromatic scale going from one *do* to the next *do*, one will tune the notes in the following order, according to these successive intervals : *(Figure 2)*

1. starting from note *sib* (remember that in Mediaeval times this term did not apply to a fixed note)
2. tuning the *fa* by a natural descending fourth (not by an ascending fifth, which would make one go beyond the octave in this scale)
3. tuning the 2 *do*'s of this scale an octave apart, by an ascending fifth followed by a descending octave
4. tuning the *sol* by ascending fifth
5. Tuning the *re* by descending fourth
6. Tuning the *la* by ascending fifth
7. Tuning the *mi* by descending fourth
8. Tuning the *si* by ascending fifth
9. Tuning the *fa#* by descending fourth
10. Tuning the *do#* by descending fourth
11. Tuning the *sol#* by ascending fifth
12. tuning the *re#* by descending fourth

figure 2

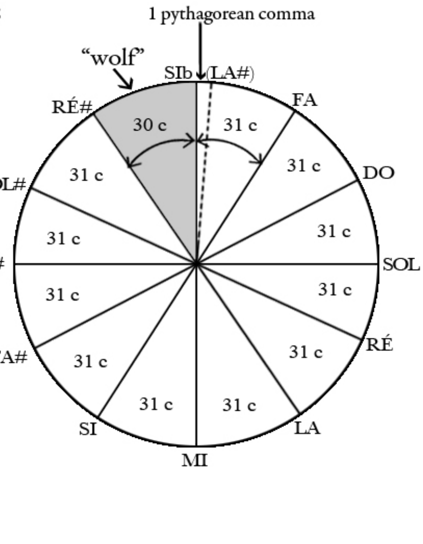
Fig. 2



(1) Natural 4th (22 commas) (2) Pure 5th (31 commas) (3) Natural octave (53 commas) (4) "Wolf fifth"

Whereas the 11 ascending fifths and descending fourths are all rigorously natural (without beats), the remaining fifth, the 12th : *re#/ sib*, "screams" because its Pythagoric comma has been amputated, which can only have made our mediaeval ancestors wince… Yet this didn't hamper musicians at the time because one could easily avoid playing that fifth. Indeed between the C13th and the C17th, music had few modulations and was only just beginning a slow evolution from modal music (but without modulations in the modern sense of the word) to tonal music, which opened the way to modulating - or rather, "tonulating", as Jacques Chailley suggests -, in every tone ;
this requirement was only met at the end of the C17th and the early C18th.

Fig. 3



However, until the C16th, the Pythagoric approach to tuning in pure fifths led people to regard it as **impossible to keep all the pure fifths on a keyboard limited to 12 sounds per octave** since it seemed to lead to a inevitable "wolf-fifth".

This tuning technique and the resulting observation are clearly set out in the well-known "Circle of Fifths" which theoricians went on researching in order to work out new temperaments from the C16th onwards.

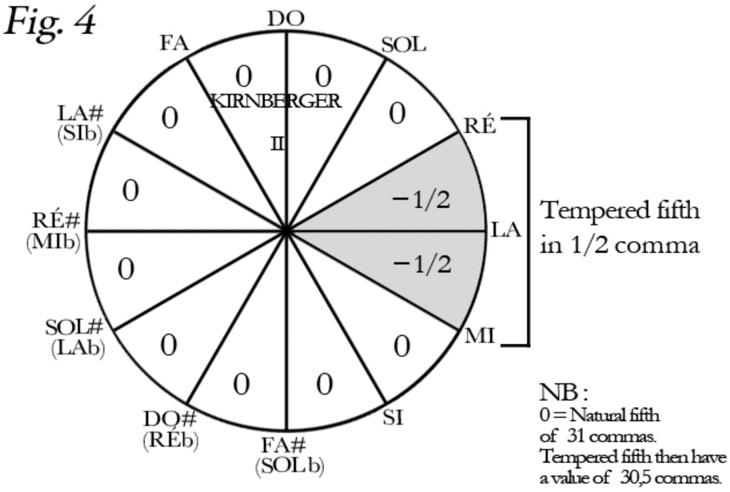
figure 3 : This is how Pythagoric tuning is presented

By examining this circle, which appears to contain a cycle of 12 Fifths **within a single octave,** in accordance with the practice of alternating ascending fifths with descending fourths which allows for dividing an octave in the way we have seen, we necessarily have to shorten the 12th fifth drastically if we wish to utilise a maximum number (11) of pure fifths. But, from the C18th onwards, people wanted to be able to modulate in every tone on the keyboard, **just as they were already doing in the orchestra,** where the number of notes was not limited to 12.To achieve this, the "wolf-fifth" absolutely had to disappear, which meant amputating that fifth by a whole comma.

All the so-called baroque "transition" temperaments, meaning in transition from the antique temperaments (primarily the Pythagoric temperament and then the temperament known as "meso-tonic" of the C16th and C17th) and the equal temperament, will thus tend to efface this missing Pythagoric comma that only affected one fifth, making it untrue and blocking the access to 6 tones out of 12.

Instead of making one single fifth bear the consequence of this amputation, they thought of spreading it over several fifths. KIRNBERGER**,** for instance, instead of shortening a single fifth by a whole Pythagoric comma, shortened two successive fifths by a half-comma. This is known as tuning to KIRNBERGER II (*figure 4*).

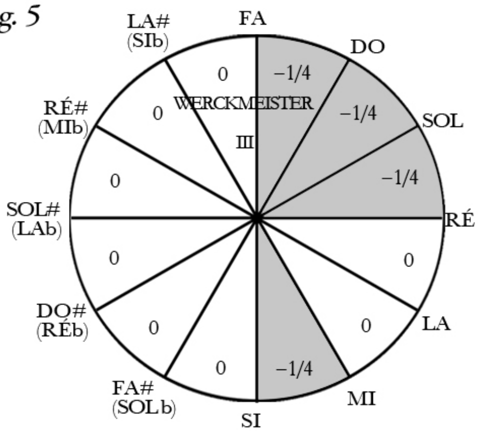
Fig. 4



This tuning apparently did not have much success, since even a fifth shortened by half a comma sounds very out of tune !

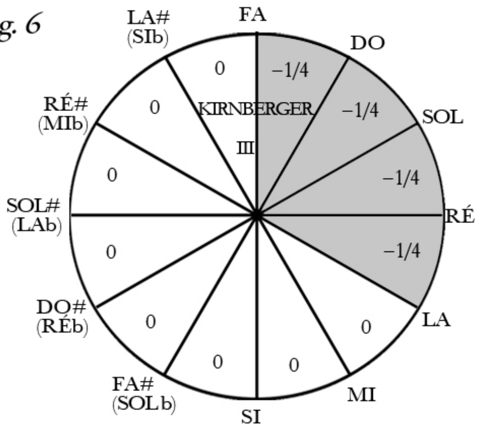
On the whole, people usually chose to shorten 4 fifths by a quarter of a comma - each person selecting which fifths to shorten : this gave rise to a great diversity of solutions. The best known of these is the tuning to WERCKMEISTER III, in which the altered fifths do not follow one another :
figure 5

Fig. 5



And KIRNBERGER III in which, on the contrary, they do follow one another :
figure 6

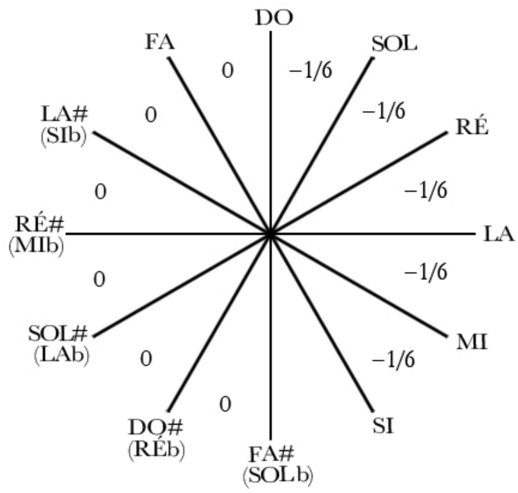
Fig. 6



These choices depended on the theoricians' aesthetic criteria (not always in line with the composer's criteria) of which a foremost concern was to preserve a certain number of natural thirds. There were also temperaments which shortened 6 fifths by 1/6th of a comma : *figure 7*

Tartini-Valloti tempérament
(mid XVIIIth century)

Fig. 7 overall diagram



In the end, the very number of the suggested solutions led to their ruin ; none of these systems really held sway. None became a lasting model of perfect tuning in the way that the Pythagoric temperament had been in the Middle Ages, the mesotonic temperament was in the C16th and C17th, or the classic equal temperament and PEFT have been since the C18th. There are many reasons for the precarious lives of these transitional temperaments also known as unequal temperaments, and I shall not go into them here. Let us simply say that their main failing was that they did not allow composers full freedom in modulation ; this is evident when one examines the tonalities employed in organ works by J S Bach, - as opposed to the total freedom observed in "the Well-Tempered Keyboard" for instance.

From another standpoint, the very diversity of these temperaments proved to be a serious handicap to achieving perfect tuning of instrumental ensembles that included a keyboard instrument, because none of them altered the same fifths ! Obviously, the orchestra couldn't handle this : it seems that stringed instruments had always continued to TUNE IN PURE FIFTHS - as QUANTZ reports in his writings on the orchestra and the tuning of violins.

So, despite opposition from theoricians who were very partial unequal temperaments for varying reasons, such as maintaining natural thirds, and despite the technical difficulty it presented, - especially on a piano -, the equal temperament did in the end manage to impose itself.

Coming in a straight line from the logic behind the baroque transition temperaments, the Equal Temperament put forward by WERCKMEISTER, MARPBURG and then J. S. BACH in Germany and by RAMEAU in France was quite simply to extend the practice of shortening fifths from just a few fifths to all 12 fifths in the circle, by amputating each one by 1/12th of a comma :

Fig. 8

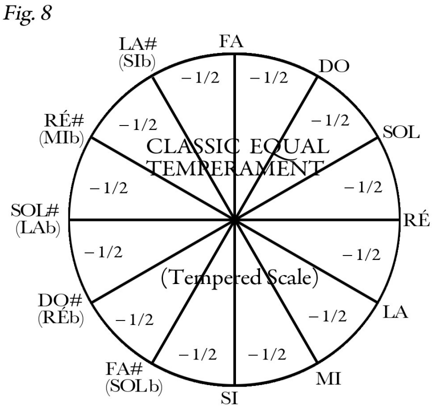


figure 8

For this last redistribution there seemed to be **only one solution**, not several, such as spreading the amputation of the Pythagoric comma over only 4 fifths selected from among the 12 existing fifths ; what is more, that solution at last provided the sought-after total freedom of modulations without exceeding 12 notes per octave. The price to pay seemed minimal : a slight alteration of the purity of each and every fifth, and a noticeable alteration of the thirds, but which in the end was less noticeable than in the unequal transition temperaments (KIRNBERGER II) in which, out of 12 thirds, 7 were more noticeably out of tune than in the Equal Temperament, even Pythagoric or semi-Pythagoric thirds.

This Equal Temperament that gives 12 tempered fifths gradually took over under the name : "Well-Tempered Scale" ; It seems that this name was originally given to all the temperaments which in principle gave access to every one of the tones (viz KIRNBERGER III or WERCKMEISTER II) and was soon carried over to the Equal Temperament alone, as the sole survivor ! Little by little the name in fact got simplified to "the Tempered Scale, although this is somewhat inaccurate but does show that from the 19th on, only one solution was adopted for tuning keyboard instruments, and that all the others had been forgotten !

Following this triumph of the Equal Temperament, first named "Well-Tempered Scale" and then "Tempered Scale" on keyboards, musicologists and theoricians tended to take into account two systems only ; indeed, they opposed them to each other : the TEMPERED Scale on the piano or the organ, which was convenient but also out of tune, using tempered fifths among other things, on the one hand - and on the other, the TRUE Scale, that is to say the vocal or orchestral scale regarded as the only true one, in particular on account of its perfect fifths.

I myself would prefer to designate the Tempered Scale as "**Equal Temperament with Tempered Fifths**" as opposed to the "**Equal Temperament with Pure Fifths**" which I am presenting here, because there are in fact at least two forms of dodecaphonic equal temperaments in existence.

On close examination, we note that the result of the Equal Temperament strategy of shortening all of the fifths by 1/12th of a comma resulted in making the sum of 12 fifths the same as the sum of 7 octaves ; this was a *sine qua non* condition for merging the enharmonic notes : in the example we have seen, the last note of the cycle of 12 fifths, *la#* merges with the 1st note, *sib.*

If we stick to our usual logic starting from the tuning technique employed, - i.e. over a single octave ands within the framework of the “circle of fifths” -, this is the only solution open to one : whatever you do and however you calculate it, if, at the end of 12 operations alternating fifths and fourths, you want to hit upon the starting *sib* - or the one an octave above -, then you necessarily have to amputate one comma from the full range of fifths, - be it by taking ½ comma from 2 fifths, ¼ comma from 4 fifths, 1/6 th comma from 6 fifths, or the one finally adopted : taking 1/12th comma from all 12 fifths.

Yet this kind of calculation is inaccurate because, following this procedure and based on the logic of this circle, one gets the impression that there are 12 fifths forming a single octave, since the initial *sib* at the top of the circle merges with the final *sib* : there appears to be only one *sib.* If you then want the initial *sib* to overlap the final *la#,* then there appears to be only one solution : that of shortening all of the fifths by 1 comma.

Another solution might suggest itself - but can only be dropped - which would be to make the final note of the cycle not the upper *sib* a perfect octave above the initial *sib*, but the upper *la#* just one comma above it. In that case we would have to extend the *sib/ sib* octave by a whole comma (see E. LEIPP, "Acoustique et Musique" p 140-141), which would prove just as much of a handicap as shortening one single fifth by a comma, for a fifth lengthened by a whole comma is just as much out of tune as is a fifth shortened by the same amount : one would merely be replacing one "wolf" by another…

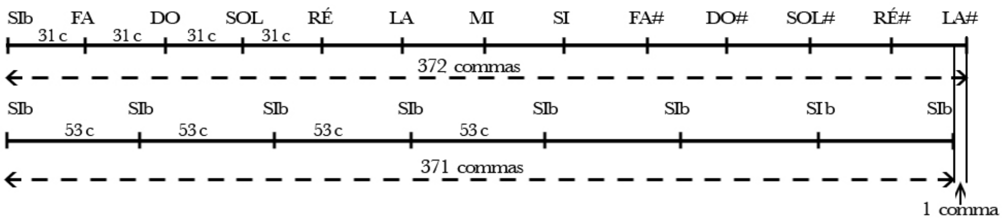
But in fact this circle we've been using is a vicious circle : the final *sib* (see *fig 3)* does not merge with, nor is it an octave above, the initial *sib,* - as the contortion of the circle might suggest. In reality this final *sib,* which on the circle of fifths overlaps the initial *sib,* lies 12 fifths higher, - i.e., 7 octaves higher. In order to represent the musical reality instead of basing oneself on a false figure, one needs to actually line up the 12 fifths alongside the 7 octaves (*fig 9a*), and then one can see that there are not one, but **two** solutions to the challenge of equalising.

One can indeed manage to make the sum of 12 fifths equal to the sum of 7 octaves by shortening all of the fifths by 1/12th (*fig 9b*) - the sole solution obtained by the technique of tuning alternate ascending fifths and descending fourths based on the logic of the circle containing all 12 fifths.

But one can also adopt a solution completely obscured by the use of the circle and the ancient tuning techniques, which is to maintain all the pure fifths and extend each octave by 1/7th of a comma (*fig 9c*) : this is the solution I have called the "Pure Fifth Equal Temperament"

Figure 9 - The correct way to set out the problem of equalisation

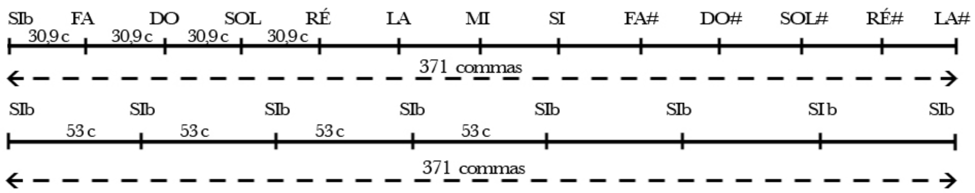
a) the acoustic data



To make a temperament equal, you need 12 fifths to equal 7 octaves

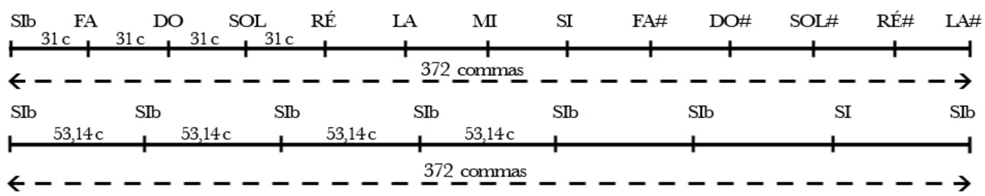
b) 1st solution : the tempered scale

You shorten each fifth by 1/12th comma



c) 2nd solution : pure fifths equal temperament

You lengthen each octave by 1/7th comma



You can see why those who researched temperaments were blinded by the procedure they used - ascending fifths and descending fourths - and the logic based on a circle of fifths within a single octave. By overlapping the starting note and the final note of the cycle, the circle of fifths presupposes that an octave can only be “natural” and thus renders it untouchable. In this concept, each note a natural octave away (a factor of 2) seemed to overlap and bear the same name. Yet this is a misconception and does not correspond to the way we perceive this sound-phenomenon : since the introduction of electronic tuners based on the natural octave (i.e. based on the misconception I have pointed out) we well know that treble sounds placed 2 or 3 natural octaves above the corresponding sounds on the instrument (medium) appear to be far too low. This justifies, - in a qualitative way at least -, the "Mels scale" which demonstrates that the higher we go into the trebles, our ear actually hears these theoretically accurate sounds as being flatter and flatter.

In other terms, the second solution, the PFET seems closer to our perception than does the traditional solution ! But before embarking on a more detailed examination to find other explanations for the superiority of this second solution, I must say right away that though it is better, it nevertheless remains INADEQUATE as far the ear is concerned. Though in my opinion the PFET is better suited to the bass and middle ranges than the Tempered Scale is, it still produced sounds in the treble that are far too low : this is as patently obvious for sounds that have no harmonics, like the flute, as it is for the treble keys on the piano (which also have poor harmonics). In this critical range, the auditory curve levels out in an exponential way (it would be interesting to verify to what extent it varies according to age and individual characteristics*)*. So PFET may be better adapted to our perception yet fall short of what the ear expects as far as the high trebles are concerned. Does this imply we need to extend the octaves even further and produce "reversed tempered fifths" i.e. extended but less true ? I do not believe so ; I rather think we should stick to PFET in the bass and middle ranges, and only progressively extend the octaves and fifths in the context of a kind of "expanding temperament" from the *mi4* that corresponds to the top E-string or chanterelle on the violin. Why?

Because I believe that the tuning which corresponds to our present-day sense of well-in-tune stems from being accustomed to the orchestral scale that our ears have been hearing for a long time. Now this sense of "well-tuned" can only correspond to the PFET up to the violin mi*4* : it necessarily registers as such from the bass to the top *mi4* in the chain of perfect fifths that corresponds to the way stringed-instruments are tuned (excepting double bass) so there is no question of expanding any fifths below this upper limit : up to the chanterelle/top string of the violin, our "accoutumance" to the tuning of stringed-instruments has set a kind of fail-safe limit to intonation correctness : the cycle of pure fifths, - *not* tempered fifths! Beyond the mi*4* of the chanterelle, there is no built-in limit, and the mechanisms of perception relative to the physiology of the ear no doubt take over, progressively stretching each interval.

It remains for me to prove that, below that *mi4* of the chanterelle, it is the PFET which prevails, both on the piano and in the orchestra, not the Tempered Scale nor the Pythagoric scale - which has its own cycle of pure fifths and is easy for the orchestra, leaving the piano to simply follow it ! Nearly all present-day musicologists admit that the spread of the Equal Temperament from the C18th onwards was not due to keyboards alone, neither did it only affect compositions for the piano but musical composition in general, for the orchestra as much as for the piano. Hence, for composers, Equalisation meant that - a *re#* could now be regarded as a *mib* and treated as such. The full ambiguity of tones, which later is to overhaul harmony and, in the long term, pull apart tonality, is constantly present both on the piano and the orchestra from the C18th on. This then presupposes a single concept when composing, be it for the piano or the orchestra, and this concept was based on being accustomed to the exclusive use of the Equal Temperament.

But beware! Don't make me say what I did not say- as has sometimes been the case ! I did *not* state that orchestra musicians played STRICTLY IN EQUAL TEMPERAMENT ! In practice, there is a constant play of attractions and repulsions that are tonal, harmonic, or "rubbing together" and these are what give life to instrumental performances. I merely said that from then on, any orchestral work was conceived in the Equal Temperament on account of the ear being accustomed to the way the piano was tuned ; if that weren't the case, an orchestral work transposed onto the piano would sound out-of-tune! Yet this is not the case if the piano has been tuned correctly, i.e. not only to Equal Temperament but in Pure Fifths (excepting the top trebles). This hypothesis, which my own ear confirms, that the orchestra itself also plays in Equal Temperament is also confirmed by various statements, such as :

Alain DANIELOU , in "L'Encyclopedia de la Musique" Directed by Igor Stravinsky (ed Fesquelle), published in 1959, writes: "The scale now used in music is the well-tempered chromatic scale"

Serge GUT, in "Sciences de la Musique" Directed by Marc Honegger, published in 1976, who professes the same belief whilst also noting that singers and violinists "always tend instinctively towards Pythagoric values" (but the PFET indeed presents many Pythagoric values since it uses perfect fifths, - which fully justifies S. Gut's observation and my own thesis !)

Harry HALBREICH, in "La Musique", an Encyclopedia directed by Maurice LE ROUX, published 1979, states :

"We see an evolution in the upper reaches towards more and more equal tones and semi-tones, leading to the Well-Tempered Scale, - that specifically European and purely artificial compromise- which allowed the most gigantic constructions of universal music, from the "Well-tempered Keyboard" to the "Tetralogy"… One can say that the two great centuries of tonal music - the C18th and C19th - were able to develop once Werckmeister's Equal Temperament had opened the way to complete freedom in music writing exploration. The Well-Tempered keyboard thus acquires the status of a manifesto…"

It is perfectly obvious that for these musicologists, the Tempered (or 'Well Tempered') scale and the "Equal Temperament" were one and the same thing, since "Equal Temperament" was synonymous with "Tempered Scale" and implied tempered Fifths. The hypothesis that there might exist an Equal Temperament with Pure Fifths had not even been considered, because it seemed to belong to the realm of Utopia because it was impossible to reconcile with the "Circle of Fifths" ; Yet, if the orchestra plays well, given minor tonal drifts for expression needs, in Equal Temperament, that can obviously only be the Pure Fifth Equal Temperament, since the Equal Temperament fits into the pure fifths, but not into the tempered fifths, despite what the theory proclaimed.

So, musicians are in reality conditioned to using the PFET scale and not the tempered scale, …with the exception of pianists, at least when their piano tuner - as is still often the case - remains constrained by the traditional equal temperament theory which requires the fifths on the piano to be tempered. This persisting belief is not actually much of a bother for a pianist who plays by himself in isolation : he gets accustomed to the tempered tuning mode… But it becomes far more bothersome when the piano is associated with the voice, with other instruments, or with an orchestra. Then, the piano sounds out of tune, too flat… even for the pianist himself. So it is quite normal that in order to avoid justified criticisms, piano tuners preferred to restore the orchestral way of intonating to the piano. This is a fair return, in fact, as the piano had imposed its own Equal Temperament on the orchestra.

To conclude, I should like to go back to certain arguments employed against the PFET, which have led me to refine my creed, and heighten my awareness of other major delays in musical theory in relation to actual practice.

People have several times made this comment : "You re-establish the pure fifths, but in return you falsify the octaves by stretching them by 1/7th of a comma. So, you gain on the swings what you lose on the roundabout : anyhow, trueness can only be a compromise…" This is so, but only if one limits oneself to the level of the “naturally” true, sometimes also called “mathematically” or “acoustically” (or even “physically”) true, and one doesn't need to be a great savant to state that this trueness is not the reference of the musician when he is appraising the musicality of a performance or the tuning of a piano. In other words, the choice of correct or acceptable intonation in musical terms does not necessarily compare with the results of natural intonation.
Now in claiming that the traditional theory of the Tempered Scale falsifies the fifths whilst maintaining pure octaves, as well as that, on the contrary, the Pure Fifths Equal Temperament maintains pure fifths but falsifies the octaves, they are still referring solely to the naturally true and not the musically true ; or rather, they are still presupposing that musical trueness coincides with “natural” trueness. This grave error is, alas, kept alive by numerous manuals on musical theory still in use in our Music Schools ! Though this theory was expounded by ZARLIN along with the discovery of the “natural third” in the late C16th, - and was in practice contradicted by the end of the C17th with the early transition temperaments using even more outrageous thirds than in the Equal Temperament - this theory is still currently vogue. Since they regarded the octave of the PFET, - 1/7th of a comma short of the natural octave -, as being musically false, what then should be said of the thirds in the Equal Temperament which are short of nearly a whole comma (2/3 of a comma, to be precise) ?

What does HARNONCOURT himself have to say about our present-day thirds which are far from “natural” ? :

"For musicians today it is very hard at first to play or to sing natural thirds (that have no beats) because they are accustomed to the piano's tempered thirds and have the impression that the natural thirds are too compressed and out of tune…" HARNONCOURT : “Le Discours Musical”. Editions Gallimard 1982, p87.

This apparently simple statement is full of wisdom, clearly showing that :

1) The feeling of correct intonation - the only one that counts for the musician - is not only a question of acoustic physics or of mathematics, but is also largely the result of a process of becoming accustomed or culturally adjusted.

2) That the way keyboard instruments are tuned has played, and still plays, a prime role in educating the ear and hence, our perception of trueness.This applies to ALL musicians, as is confirmed by HARNONCOURT : when tackling the problem of judging the trueness of thirds, he refers to “today's musicians”, - not just to pianists.

Is musical trueness therefore merely a cultural phenomenon - as I once thought for a while ? I no longer believe this, because one can get accustomed to any scale, and the study of all the diverse scales and musical languages that exist or have existed shows that they all have been influenced either by natural trueness at one time or another, or else (which comes to the same thing) by “harmonic generation” to quote the title of a very interesting work by RAMEAU (in which he definitively plumps for the Equal Temperament).

But what is the reason why musical trueness cannot be natural trueness ? It lies in the fact that when using solely natural intervals it is not possible to create a stable musical scale, nor a musical language compatible with Western musical practice. Natural intervals are not mathematically compatible one with another ; the sum of 7 octaves is not equal to the sum of 12 natural fifths. You therefore have to either modify the fifth or the octave, and choose between maintaining the trueness of either the fifth or the octave (Tempered Scale - or PFET?). What is more, 4 natural fifths give rise to a Pythagoric third longer than a natural third by a whole comma : here too one has to choose. Pythagoras chose to sacrifice the third in favour of the fifth ; but contrarily, with the invention of the “mesotonic temperament” the early Baroque musicians sacrificed the fifth in favour of the natural third, having discovered that interval on the monocords.

So we clearly see that, whilst starting from sounds naturally produced, these had to be modified to construct viable scales and enable coherent musical transposition, just as a mason chips (tempers) his bricks so as to ADJUST them - what an apt word ! In the end the only “true” sound material is the one that has been adjusted, since no natural scale composed solely of natural intervals could be made to exist. In this respect, Zarlin's so-called “natural” scale, still put forward as the model for the Western scale in our Solfege manuals and Encyclopedias, is a veritable imposture because it is simply impossible, both mathematically and musically : among other things, in its second interval, it includes a real "wolf-fifth" amputated by one comma, which means IT CANNOT EVEN BE PLAYED IN *DO* MAJOR! But apparently this is a weak degree! Just make a musician listen to it and then ask a violinist to tune his *re/la* fifth in this way! Only an unrepentant mathematician unable to hear the interval of one comma could live with such an interval.

So it really is a question of “adjusting” the intervals, which explains temperaments. But of course this adjustment has varied with instrumental practice and musical composition according to particular epochs and civilisations : hence the existence of different scales and systems throughout the world. In the last two centuries, the only temperament that completely fulfilled the requirements of musical practice (tuning orchestral instruments in pure fifths, on the one hand, and the widespread use of the Equal Temperament, on the other) had to be the Pure Fifths Equal Temperament (PFET). This is why musicians and tuners have long since put it into practice, and why to our ears, which have long been accustomed to it, the intervals it contains sound particularly true - despite not being natural, just as the octaves and the fifths aren't natural.

Thus, following HARNONCOURT, I am able to state that – TO OUR PRESENT-DAY EARS - natural thirds sound false because they are too short. And I shall add that, - albeit to a far lesser degree - the same holds true for natural octaves, as is proven by a piano tuned strictly in accord with the well-tempered scale, i.e. based on dividing the natural octave into 12 strictly equal semi-tones.

There is another objection raised in opposition to the use of PFET which seems more interesting : it recognises the fact that on a well-tuned piano, the pure octave is not the natural octave, but an octave expanded by the fraction of a comma. However, it does not attribute this expansion to a cultural factor, as I do with PFET, but to a physical phenomenon: the inharmonicity of the piano strings.

What, then, is this inharmonicity - which indeed plays a great role in piano tuning ?

We know that by definition a perfect octave corresponds to a 2:1 ratio of frequencies. The sense of trueness here, just as 2:3 is for a fifth, is due to the absence of beats, which is what characterises a natural interval. For indeed, if an octave corresponds to the 2:1 ratio, its that the 2nd harmonic of the bass note, having twice the frequency of the fundamental note, will in theory be found at the same height as, and thus overlapping, the top note of that octave ; there will be no beats, and thus the octave is said to be “pure” or “natural”.

But in reality, this is not what happens : the strings of the piano do not produce real harmonics * ***footnote 5***, but overtones, always higher than the partials produced by harmonics. Consequently, if we tune a note on the piano precisely one octave above another note by setting it at twice the frequency of the bass note, that octave will not sound "natural", but give out beats because, in this case, the 2nd partial of the bass note emits a frequency that is higher than that of the upper note, instead of being at exactly the same pitch as would be the case for the true harmonic of the lower note.

To avoid this phenomenon of making the top note seem too low, a tuner would instinctively enlarge the octave in order to place the higher note exactly level with the 2nd partial of the bass note, thus re-establishing the "natural" justness of the octave. I am in the process of studying the repercussions of inharmonicity on piano tuning, and this research leads me to think that there are major repercussions in the treble and top treble registers, and also in the lower register, in the case of certain instruments with strings that are too short.

Yet the distortions due to inharmonicity on the whole remain fairly limited in the medium register or tenor. In this register, the enlargement of the octave attempting to correct the inharmonicity remains too feeble to entirely re-establish the trueness of the fifths. This is re-established only in the trebles, where the inharmonicity is very strong and dilates not only the octaves more and more, but the other tempered intervals as well, including the fifth - which now at last is true.

The above system, whilst it is far closer to the PFET than to the tempered scale, does provide a scale which, within a given diapason, provides lower notes in the treble than those of the orchestral scale which from bass to treble is defined by pure fifths but very likely thereafter, above the *mi4* (chanterelle) top string of the violin, by dilated fifths (see below).

We still need to make experiments to discern whether this enlargement of the octave seen in both approaches and observed by all acoustic laboratories is explained by the inharmonicity of the piano strings or by the PFET theory which totally restores the real value of the pure fifths as they exist for orchestra instruments and singers.

The good news is that both theories tend in the same direction, viz : dilating the octaves and restoring the fifths toward their orchestral value (a total restoration as far as PFET is concerned). True, the explanation is different : one of the tuning systems is presented as a Tempered Scale "corrected" on account of inharmonicity, and the other, as a new theory of "trueness" for both piano and orchestra. But both result in frequencies that are fairly close yet similarly distant from the frequencies of the theoretical tempered scale. So, these two tuning systems of the theoretical tempered scale (automatically produced by electronic tuners) can easily be distinguished, but it is not so certain that everyone can distinguish between "the tempered scale corrected for inharmonicity" and PFET.

Where the distinction can be made, which form of tuning will prove to be the most true - or rather, the most musical ? Will there be clear preferences according to individuals, or according to the repertoire ? These questions can only be clearly answered one way or another thanks to the research I am undertaking with the help of pianists, musicologists and musicians. In any case, here is where the real choice lies, not between PFET and the tempered scale- the scale still taught in manuals, described in encyclopaedias ; a scale which electronic tuners have nonetheless demonstrated as being inadequate for our present-day perception of trueness. Research in to temperament and trueness must surely needs progress by shaking up ingrained ideas (reçues) : one can only be horror-struck by the unbelievable lapse of two centuries between the theory, - still promoting "natural trueness”-, and the actual practice of musicians. This time-lag has been as harmful for the practice itself as it as been for the understanding of musical notation and of its evolution.

NOTES

(1) Amongst others : Yehudi MENUHIN, Paul BADURA-SKODA, Alfred BRENDEL, Dalton BALDWIN, René VIGNOLLES, Nicole AFRIAT, Pierre-Laurent AIMARD, Roland de CANDÉ, Jacques CHAILLEY, Pi-hsien CHEN, Gérard CONDÉ, François-René DUCHABLE, Pascal DUSAPIN, Thérèse DUSSAUT, Peter ETÖVÖS, Maurice FLEURET, Henri FOURÈS, Kazuoki FUJII, André GOROG Jean-Marie GOUELOU, Jean GUILLOU, Eric et Tania HEIDSIECK, Yves HENRY, Cyril HUVÉ, Christian IVALDI, Pierre JANSEN, Irène JARSKY, Martine JOSTE, Claude LAVOIS, Yvonne LEFÉBURE, Denis LEVAILLANT, Emile LEIPP, Alain LOUVIER, François-Bernard MÂCHE, Bernard MAUPIN, Roland MEILLER, Dominique MERLET, Alain MOTARD, Alain NEVEU, Jean-Claude PENNETIER, Yves POTREL, Pierre RÉACH, Jean-Claude RISSET, Patrice SCIORTINO, Catherine SILIÉ, Fernand VANDENBOGAERDE, Colette ZERAH.

(2) Reminder: the fifth is defined acoustically by the 2:3 ratio. This ratio corresponds to the frequencies of the two notes that make up that fifth. These are the fifths used by the orchestra, especially to tune string-instruments, but the other instruments and the singers also use pure fifths and not the piano's so-called “tempered” fifths.

(3) Remember also the two ways to measure intervals. When one plays a first ascending fifth like *do/sol* for instance, followed by a second such as *sol/ re*, the ear perceives a succession, or an adding up, (conjunction, sequence ?) of two equal intervals. Twice, the ear perceives a span of the same height -the span that characterises the pure fifth. Knowing that a perfect fifth is worth 31 commas (see note 4, below). When passing from a *do* to the *re* 9 notes above it, the ear spans 31+31=62 commas. This is what we perceive. In reality two successive multiplications of the starting note frequencies (here, *do*) take place in that same 2/3 ratio characteristic of the perfect fifth.

Whenever we perceive a perfect fifth interval between two notes this is in fact because the frequency (i.e. the number of vibrations per second) of the upper note of the fifth is 1,5 times higher than the lower note frequency.

One could also express this by saying that when the ear perceives a perfect fifth interval between two notes, it's because the ratio of the frequency of the upper note to that of the lower note is equal to 2/3 or 1,5.

In the example we have taken here, the *do* frequency being 260 vibrations/second, that of the *sol* one fifth just above it will thus be 260 x 1,5=390hz, and that of the *re* one fifth above that,- i.e. 9notes above the initial do-, will be 390 x 1,5 x 1,5/260 = 1,5 x 1,5 = 2,25.

Where the ear perceives an adding up of two similar intervals (piled on top of one another) -in this case 2 perfect fifths, so 31commaas + 31commas = 62commas- what has actually happened is that the base note frequency has been multiplied twice by the same f'requency- ratio that characterises that fifth :

$$1{,}5 \times 1{,}5 = 2{,}25$$

In a reciprocal way, knowing that the ratio of a major 9 is 2,25, we are able to know of a perfect fifth since a major 9th results from two successive perfect fifths: it will that ratio
If we call that perfect fifth ratio **y** , it should be such that

$$\mathbf{y} \times \mathbf{y} = 2{,}25 \quad \text{or :} \quad \mathbf{y}^2 = 2{,}25$$

We thus see that the ratio y which corresponds to the perfect fifth will be the square root of 2,25.- in other words, that number which, twice multiplied by itself, brings us back to 2,25 ; this is written : $\mathbf{y} = 2{,}25^{1/2}$

Each interval can thus be expressed as a span (here expressed in commas) yet also as a ratio of the frequencies of the two notes that compose it. To each interval there corresponds not only a number of commas but also a ratio of frequencies. The different spans expressed in commas (or in other units of measure called "logarithmetical" such as *savarts* or *cents)* which add up or subtract from each other according to whether the frequency ratios multiply or divide themselves.

In this way the span which characterises the perfect octave may be considered as the sum of a fifth worth 31 commas and a fourth worth 22.

1 octave=1 fifth + 1fourth= 31 com. +22com = 53 commas

But the frequency ratio that characterises the octave is equal to the ratio of the fifth : 3/2, MULTIPLIED by that of the fourth 4 /3 :

Ratio of the octave = 3/2 x 4/3 = 2/1 or 2

Our aim of dividing the octave into 12 equal semi-tones can itself be tackled in two ways, depending on whether we are concerned about the different spans, or the frequencies and their ratios. What then, in either case, is a semi tone of the "classic" Equal temperament worth ?

Expressed in commas, it is quite simply the value of an octave divided by 12 :

Tempered 1/2 tone = 53 commas / 12 = 4,416666…commas

Expressed in terms of frequency ratios, this semi-tone needs to have a value **d** such that multiplied 12 times by itself it gives 2, namely, the octave ratio. We need :

$d \times d \times d \times d \times d \times d \times d \times d \times d \times d \times d \times d = 2$, that is : $\mathbf{d}^{12} = 2$

hence $\mathbf{d} = 2^{1/12}$

which is the 12^{th} root of 2, and our pocket calculator gives us

$$\mathbf{d} = 2^{1/12} = 1{,}05946\ldots$$

Knowing this figure, and knowing that the *la3* (the *la* of the keynote/diapason) is fixed/established at some diapason - 442 for instance- it then becomes easy to obtain all the frequencies of the notes of the tempered scale : you just divide 442 by **d**, the ratio of the semi-tone, in order to obtain the frequency of *sol#3*

F*sol#3* = 442 / 1,05946… = 417,192…

…and then divide the frequency of *sol#* once again by **d** to obtain that of *sol3*

F*sol3* = 417,192…/ 1,05946

…and so forth, to obtain all the frequencies of the notes below *la3*.

For the higher notes, you only need to multiply 442 by **d** to get the frequency of *la#,* and so forth for the treble notes above it.

(4) For simplicity's sake, I am taking as my unit "Holder's comma" defined as having a value of 1/53rd of a natural octave. In this case a perfect fifth (the violinists's or PFET fifth) is - allowing a excellent approximation - worth 31 commas (quite precisely, 31,003 Holder commas) and the fourth is then worth 53 - 31 = 22 commas. Simple arithmetic then permits the calculation of Pythagoras' scale in terms of commas.

(5) When we perceive a musical note such as the bass *do* of the cello, we believe we have heard a single note at the pitch perceived - here, bass *do*. In reality, this note is mostly a complex sound resulting from the synthesis of several other sounds higher than the perceived sound, called harmonic notes or "harmonics", whereas the perceived note is called the fundamental note. The "harmonics" are lined up in an unchangeable order from the bass to the treble : one octave above the fundamental we find the 2nd rank harmonic, or 2nd harmonic (rank 1, the lowest is where we find the fundamental, thus considered as the 1st harmonic…)

In our chosen example, then, we have another *do*. Let's call them *do1* and *do2*. The 3rd harmonic is the harmonic "of the fifth" because it is always a fifth above the 2nd harmonic ; so here it is a *sol*. The 4th harmonic is one fourth above the 3rd harmonic, and it's a *do* which we shall call *do3*, which we therefore always find 2 octaves above the fundamental.

The 5th harmonic is called "the harmonic of the third" lies a major third above the 4th harmonic, - here , a *mi*. The 6th is a minor third above the 5th harmonic, hence an octave above the 3rd harmonic : here, a *sol.* The 7th harmonic, known as the "natural" 7th', is a minor third above the 6th, here, a *sib.* The 8th is *do4*, the 9th is a *re* - it's the natural 9th*,* and so forth.

The low notes of instruments give out 20 to 30 audible harmonics which are less and less intense as they get further away from the fundamental.

On the table below, I have set out the first 20 harmonics of a *do*, as well as the first 20 harmonics of a *sol,* to give a comparison.

The frequencies of the harmonics are all multiples of the fundamental note frequency. If the fundamental *do* frequency is 260 *hz* (vibrations/second), that of the 2nd harmonic *do2* will be 260 x 2 = 520 *hz*, the 3rd harmonic, *sol*, will have a frequency of 260 x 3 = 780 *hz,* etc... But the harmonics of a fundamental *sol* with a frequency of 100 *hz* will thus in succession be 200 *hz*, 300 *hz,* 400 *hz,* etc... following the table here below :

TABLE OF THE HARMONICS OF TWO COMPLEX SOUNDS

HARMONIC TABLE OF TWO COMPLEX SOUNDS

RANK	NOTE	FREQUENCY	NOTE	FREQUENCY
20	mi	5200	si	2000
19	ré#	4960	la#	1900
18	ré	4680	la	1800
17	do#	4420	sol#	1700
16	do5	4160	sol5	1600
15	si	3900	fa#	1500
14	si*b*	3640	fa	1400
13	la	3380	mi	1300
12	sol	3120	ré*b*	1200
11	fa#	2860	do#	1100
10	mi	2600	si	1000
9	ré	2340	la	900
8	do4	2080	so!4	800
7	si*b*	1820	fa	700
6	sol	1560	ré	600
5	mi	1300	si	500
4	do3	1040	so!3	400
3	sol	780	ré	300
2	do2	520	so!2	200
1	DO	260	SOL	100

In principle, only sustained fundamental notes (such as instruments with bows, or else wind-instruments) give out true harmonics, meaning sounds with harmonics that are EXACT multiples of the fundamental note frequency. This is not the case with un-sustained notes such as on the piano, the harp or the guitar, nor of "pizzicato" notes on the strings. In this case the notes which accompany the the fundamental note are alwayshigher than true

harmonics : they are then called “partials”.
Here we give examples of the first 10 harmonics of a *la3* on the violin, and alongside , the first10 partials of the same *la3* played on the piano :

TABLE VIOLIN PIANO

	VIOLIN	PIANO	GAPS
	Frequencies		
	Harmonics	Partials	
10 Do#	4400	4556	2,7 commas : not far off a semi-tone !!
9 Si	3960	4073	2,2
8 La 6	3520	3600	1,7
7 Sol	3080	3133	1,3
6 Mi	2640	2672	1
5 Do#	2200	2219	3/4
4 La	1760	1769	1/2
3 Mi	1320	1324	1/4
La 4	880	881	1/10
LA 3	440	440	0 comma